

22. J. A. Barker, R. O. Watts, J. K. Lee, et al., "Interatomic potentials for krypton and xenon," J. Chem. Phys., 61, No. 8, 3081-3089 (1974).
23. K. L. Schäfer and F. W. Reiter, "Eine Messmethode für die Ermittlung des Wärmeleitvermögens bei 1100°C," Z. Elektrochem., 6, No. 9, 1230-1235 (1957).
24. L. S. Zaitseva, "Experimental investigation of the thermal conductivity of monatomic gases over wide temperature intervals," Zh. Tekh. Fiz., 29, No. 4, 497-505 (1959).
25. D. L. Timrot and A. S. Umarskii, "Measurement of the thermal conductivity of krypton to 2000°K," in: Thermal Conductivity (J. C. Ho and R. E. Taylor, eds.), Plenum, New York (1969), p. 151.
26. A. A. Voshchinin, V. V. Kerzhentsev, E. L. Studnikov, and L. V. Yakush, "Experimental investigation of the thermal conductivity of krypton at high temperatures," Inzh.-Fiz. Zh., 28, No. 5, 821-825 (1975).

## THERMAL DIFFUSIVITY OF INHOMOGENEOUS SYSTEMS.

### II. EXPERIMENTAL DETERMINATION OF THERMAL DIFFUSIVITY

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We investigated the methodical error in the measurement of the effective thermal diffusivity of inhomogeneous systems. We give the recommendation for the choice of the dimensions of the inhomogeneous specimen.

In [1] we established the conditions under which the nonstationary temperature field of a heterogeneous system can be calculated with an admissible degree of approximation by using the model of a quasihomogeneous body with effective thermal diffusivity

$$a = \lambda/c\rho. \quad (1)$$

In this paper we shall consider the problem of experimentally determining the effective thermal diffusivity. Measurements of thermal diffusivity use methods whose calculation formulas are based on the solution of the corresponding nonstationary problems for homogeneous bodies, and therefore there is a methodical error caused by the inhomogeneity of the representative element (specimen). By the error in the measurement of the effective thermal diffusivity we shall mean the difference between the value obtained from the experiment and the effective parameter determined in accordance with (1):

$$\delta a = \frac{a_m - a}{a}.$$

For systems with long-range order the problem of choosing a representative element of an inhomogeneous system consists in determining the number of elementary cells in the specimen for which the deviation of the temperature field from the field of a quasihomogeneous body will not lead to a methodical error  $\delta a$  that exceeds the admissible value. We use the following method of investigation:

- a) the behavior of the measuring experiment is simulated by a numerical solution obtained on a computer for the corresponding boundary-value problem for an inhomogeneous system;
- b) the value  $a_m$  of the thermal diffusivity obtained on the basis of the numerical solution by the calculation formula of the method is comparable to the effective parameter  $a$ ;
- c) a computer-empirical relation is constructed for determining the dimensions of a representative element.

The error in the measurement of the effective thermal diffusivity depends on the method of measurement, the structure of the system, the concentrations of the components and the

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relations between their properties, the ratios of the characteristic dimension of the specimen (the "macrodimension") to the characteristic dimension of the inhomogeneities (the "microdimension"). In this paper we solve the problem for two very simple two-component systems consisting of plates perpendicular and parallel to the heat flux; for brevity, we shall hereafter call these system A and system B, respectively (Fig. 1 and Fig. 3 of [1]).

The numerical solution of the problem was carried out by a finite-difference method using an implicit scheme for system A and a locally one-dimensional scheme for system B [2]; the error in determining the value of  $a_m$  was 3-5%.

As the admissible value of error in the measurement, we used the quantity  $(\delta a)_M = 10\%$  and determined the condition under which  $|\delta a| \leq (\delta a)_M$ . We give below the results of the investigation for three methods of measuring the thermal diffusivity.

1.  $\alpha$ -Calorimeter Method for a Regular Regime of the First Kind [3]. The calculation formula of the method has the form

$$a_m = km_\infty,$$

where  $m_\infty$  is the rate of cooling as the heat-exchange coefficient  $\alpha \rightarrow \infty$ ;  $k$  is the coefficient of the shape of the body for a plate of thickness  $l$ :

$$k = l^2/\pi^2.$$

We obtained the following recommendations for the choice of the dimensions of the representative element.

System A. For any combination of properties of the components which lie in the intervals  $\beta = a_2/a_1 = 10^{-3}-1$ ,  $\nu = \lambda_2/\lambda_1 = 10^{-3}-10^2$  when the number of plates in the specimen is  $N \geq 6$ ,  $|\delta a| \leq 10\%$ .

System B. For  $\beta = 10^{-1}-1$ ,  $\nu = 10^{-2}-10^2$  the "micro-macrodimension" ratio is  $\epsilon = h/l \leq 0.2$  (Fig. 3 in [1]); for  $\beta = 10^{-3}-10^{-1}$ ,  $\nu = 10^{-3}-10^{-1}$

$$\epsilon_{\parallel} \leq 0.34 + 0.067 \lg \nu + 0.092 \lg \beta + 0.0175 \lg \nu \lg \beta.$$

2. Method of Linear Heating [3]. The calculation formula for a plate of thickness  $l$  has the form

$$a_m = \frac{bl^2}{8\theta},$$

where  $b = (dt/d\tau)$ ;  $\theta$  is the temperature drop between the surface and the central cross section.

System A. For  $\nu = 10^{-3}-10^2$ ,  $\beta = 10^{-3}-1$   $N \geq 6$ .

System B. For  $\beta = 10^{-1}-1$ ,  $\nu = 10^{-1}-10$ ,  $\epsilon_{\parallel} \leq 0.1$ ; for  $\nu = 10^{-3}-10^{-1}$ ,  $\beta = 10^{-3}-10^{-1}$

$$\epsilon_{\parallel} \leq 0.176 + 0.036 \lg \nu + 0.051 \lg \beta + 0.011 \lg \nu \lg \beta.$$

3. Method of the Instantaneous Plane Source [3]. The calculation formula is

$$a_m = x^2/2\tau_M,$$

where  $\tau_M$  is the time from the moment of action of the instantaneous source until the maximum temperature is reached at a distance  $x$  from the probe.

System A. The number of plates between the source and the heat collector can be estimated by the formula

$$N \geq \frac{15|1-\nu|}{1+\nu}.$$

System B. For the choice of the ratio  $\epsilon_{\parallel} = h/x$  we obtained the following recommendations: for  $\beta = 10^{-1}-1$ ,  $\nu = 10^{-1}-10$ ,  $\epsilon_{\parallel} \leq 0.1$ ; for  $\beta = 10^{-3}-10^{-1}$ ,  $\nu = 10^{-3}-10^{-1}$

$$\epsilon_{\parallel} \leq 0.170 + 0.034 \lg \nu + 0.044 \lg \beta + 0.009 \lg \nu \lg \beta.$$

In the case of a system with a more complicated structure, combining elements parallel and perpendicular to the flux linking the components, for the choice of the ratio  $\epsilon$  of the dimension of an elementary cell to the characteristic dimension of the specimen we can recommend, from qualitative considerations, an estimate of the form

$$\varepsilon \leq \min\{\varepsilon_{\parallel}, \varepsilon_{\perp}\}, \quad \varepsilon_{\perp} = \frac{1}{N}.$$

#### NOTATION

$\alpha$ ,  $\lambda$ ,  $c\rho$ , effective thermal diffusivity, thermal conductivity, and volumetric heat capacity of a quasihomogeneous body;  $\alpha_i$ ,  $\lambda_i$ , thermal diffusivity and thermal conductivity of the  $i$ -th component;  $\alpha_m$ , value obtained by measuring thermal diffusivity of the inhomogeneous specimen;  $\delta\alpha$ , relative methodical error of the measurements;  $l$ , thickness of a flat specimen;  $N$ , number of plates in the representative element of system A;  $h$ , characteristic microdimension of system B;  $\varepsilon_{\parallel} = h/l$ ;  $\nu = \lambda_2/\lambda_1$ ;  $\beta = \alpha_2/\alpha_1$ .

#### LITERATURE CITED

1. G. N. Dul'nev and A. V. Sigalov, "Thermal diffusivity of inhomogeneous systems. 1. Temperature-field calculation," *Inzh.-Fiz. Zh.*, **39**, No. 1, 126-133 (1980).
2. A. A. Samarskii, *Theory of Difference Schemes [in Russian]*, Nauka, Moscow (1977).
3. G. M. Kondrat'ev, *Thermal Measurements [in Russian]*, Mashgiz, Moscow (1957).

#### GENERALIZATION OF DATA ON THE DEPENDENCE OF THE COORDINATION NUMBER ON THE POROSITY IN FILLINGS OF SINTERED OR PRESSED GRANULAR MATERIALS

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An approximation equation is proposed for estimating the mean number of contacts with a particle in granular systems.

The dependence of the "coordination number"  $N_c$  (the average number of contacts with a particle) on the porosity  $P$  in free fillings of sintered or pressed granular materials has been given considerable attention in publications on general problems of physics and geometry [1-3], powder metallurgy [4-6], chemical technology [7, 8], the theory of heat and mass transfer [9-14], and the properties of alloys [15, 16].

For an analytical estimate of the value of the coordination number semiempirical and purely empirical approximation relations are employed [1, 4, 5, 8], which agree quite well with experimental data, as a rule, only over a narrow range of variation of the porosity  $0.3 \leq P \leq 0.5$  (Fig. 1).

The use of existing relationships [1, 4, 5, 8] to estimate the value of the coordination number for other values of the porosity leads to considerable disagreements with the results of measurements, or, in general, leads to absurd results (zero or negative values of the coordination numbers). The latter is obviously due to the fact that in the publications mentioned the suitability of the theoretical relations employed was verified by comparison with experimental data over a small range of variation of the porosity, limited by the framework of the problems considered.

A direct geometrical solution of the problem of determining the coordination number which agrees well with experiment over a quite wide range of variation of porosity  $0.26 < P < 1.0$  (see curve 21 in Fig. 1) was proposed in [10]. If we analyze existing measurement results, we find a quite clear relationship between the change in coordination number  $N_c$  and the porosity.

Practically all the experimental points (29 out of 32) can be generalized quite well by the following approximating relationship (see curve 22 in Fig. 1):

$$N_c = [10 \sin^3 0.5\pi(1 - P) + 3] \pm 1, \quad 0 \leq P < 1, \quad (1)$$

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